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Restoration and balance of a folded and faulted surface by best-fitting of finite elements: principle and applications

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Abstract—A computer program is presented which allows us to test the restoration of a folded and faulted thin competent layer and then to balance this surface. The balance of such a surface is useful both to constrain the three-dimensional shape of the folds and the geometry of the limits of the faults. If a part of the surface is fixed the restoration can also give the finite displacement field linked to the deformation of the layer. The principle of the method is given and its accuracy is tested for the restoration of an experimentally folded sheet of paper. Finally the applicability to the restoration of natural structures is discussed.

INTRODUCTION

WHEN geological structures are drawn in cross-section, the available observations are commonly so scattered that interpolation is needed between these data. Balanced geological cross-section construction techniques are very useful to help such interpolation since this method invokes the application of some simple rules, including the restorability of the structures and the preservation of the rock volume before and after the deformation (Dahlstrom 1969, Hossack 1979). One of the most restricting assumptions for the construction of balanced cross-sections is that of conservation of volume (Goguel 1952). The simplest way to satisfy this assumption, when balancing a section, is to preserve the length of the layers and/or the area between these layers, before and after deformation (plane strain assumption). Volume change may be integrated when balancing a section, but such measurements are very rare (Hossack 1979, Mugnier & Vialon 1986).

Using the rule of conservation of area and length in cross-sections several authors have developed computer programs for section balancing (e.g. Groshong & Usdansky 1986, Jones & Linsser 1986, Kligfield *et al.* 1986, Medwedeff & Suppe 1986). In recently developed programs, interpolations made on the sections may be fitted to the geologic map (De Paor 1988), and a complete set of restoration methods can take into account most of the complicated structures of real data (Moretti & Larrere in press). Because the rule of conservation of length and area generally means that cross-sections must be drawn perpendicular to a plane of no finite extension, this limits the application of the balanced section methods.

Two approaches can be used to help in the interpretation of folded and faulted layers that have some out-of-the section material movement. These two approaches, which may rely on map view restoration, are the following.

(i) Method of removal of finite deformation using strain trajectories was suggested by Schwerdtner (1977) and Cobbold (1979), and applied to regional analysis by Gratier *et al.* (1989). Finite strain values and strain trajectories of the sedimentary cover of the French Chaînes Subalpines was deduced from the strain measurements of a folded and faulted competent layer. This deformation was removed by a finite element analysis (84 elements of about 15 × 15 km of initial size) and the displacement field of this cover was calculated. This method has at least two limitations. Firstly it covers a rather large area (the deformation is assumed to be homogeneous within each element), and secondly it needs good data on the location of the décollement surface (to estimate the vertical displacement values).

(ii) Another approach, presented here, is to test by a trial and error method, the possibility of restoration of a folded and faulted surface (Fig. 1a). The method describes a thin competent folded sedimentary layer (zone A, Fig. 1b) using rigid elements the size of which depends on the curvature of the surface. The elements are then laid flat (and automatically fit) to form a horizontal surface (initial state of the layer, Fig. 1d). The degree of compatibility tests the reliability of the geometry of the deformed layer. If the thickness of the layer does not change during folding and faulting, a plausible interpretation must be perfectly retrodeformed, as a folded and torn sheet of paper may be smoothed with an iron.

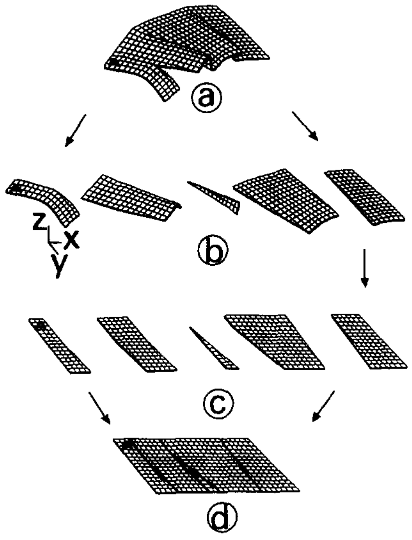

Fig. 1. Sketch of a solution to balance a folded and faulted layer (perspective view). A thin competent layer is restorable to an initial horizontal state if its deformation occurs without change of thickness, and if this layer is well drawn. (a) Deformed state, folded and faulted layer. (b) Partition of this folded and faulted surface into several zones (five) in order to have a single valued mathematical function between the X, Y and Z values of co-ordinates on the surface (after interpolation between spaced data). (c) Unfolding of each zone (limited either by faults or vertical strata) by the computer program. (d) Fitting of these zones to reconstruct the initial (horizontal) state of the layer.

PRINCIPLE OF THE METHOD

In order to restore and balance a folded and faulted surface it is first necessary to describe this surface by a network of points with X, Y and Z co-ordinates. As mentioned above the initial data set generally requires interpolating between spaced observations. Within our computer program, cross-sections or maps are digitized on a digitizing tablet, and the interpolation is done using a cubic spline function (graphical program GREG, Guilloteau & Valiron 1986), on a SPS9 Bull computer (Unix system).

A substantial problem occurs when the same layer exists several times on the same vertical line (e.g. overturned limb of a fold, thrust fault). No graphics program available to us deals with this problem. In this case it was decided to divide the surface into several zones (see Fig. 1b), so as always to have a single valued relationship between the X, Y values of horizontal co-ordinates and the Z vertical value.

After interpolation a regular XY grid is obtained for each folded zone (limited either by vertical strata or by faults, Fig. 1b). Then, it is possible to describe such a folded surface by a network of rigid triangular elements (Fig. 2a).The size and the shape of these elements depends both upon the number of points allowed in the regular grid and upon the curvature of the surface. A

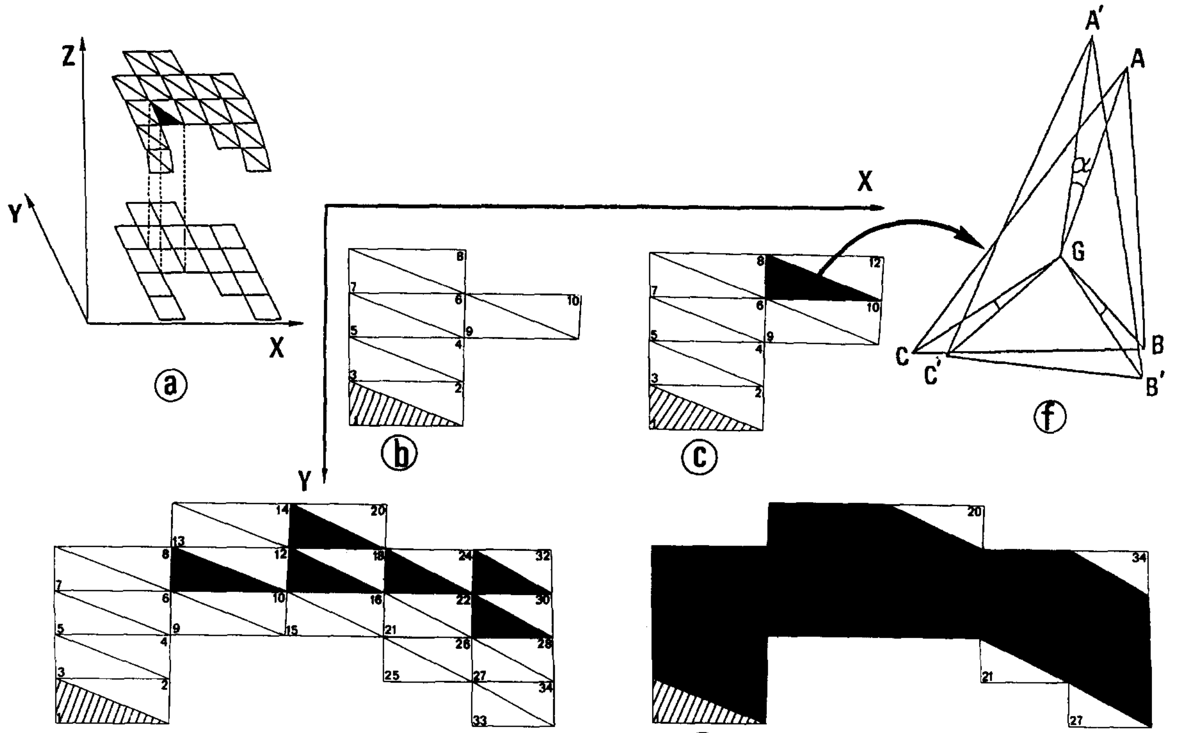

Fig. 2. (a) Segmentation of the folded surface into triangular elements generated from points on a regular XY grid. (b)–(e) All the triangular elements of the folded surface are laid flat and automatically fitted on a horizontal surface: a sketch of the sequence of fits is given, the number in each triangle indicating its rank for fitting. There are various types of elements depending on the sequence of restoration relative to their neighbours: the first (hatched) triangle is arbitrarily fixed; the white triangles, with two common vertices with their neighbours, are fixed against a mean position of the preceding elements; the shaded triangles, with three common vertices with their neighbours, are fitted into a triangular hole with the minimization algorithm (f). A first fitting iteration is done (b–d), then successives iterations are run until a minimum value of the sum of the distance between all the triangles is obtained (e). (f) Fitting a triangular element ABC in the triangular hole A′B′C′ defined by its neighbours. The following definitions are used: mA,mA′,mB,mB′,mC,mC′, are the length of the medians from G (the coincident centers of mass of the triangular element and triangular hole), α = angle AGA′, β = angle (B′GA′−BGA), γ = angle (A′GC′−AGC).

very good fit may be obtained between initial data and finite elements surface (see application of Fig. 3) if a sufficient number of elements are used.

The next step is to lay flat and automatically fit all the triangles on a horizontal surface. An example of the sequence of restoration is given in Fig. 2. From its deformed state (parallel to the folded surface, Fig. 2a) to its final horizontal state (Fig. 2e), each triangle is translated and rotated three times in order to have its three vertices in the horizontal surface. Then, various types of triangles must be considered (see Figs. 2b–e). The coordinates of the first triangle of the first column of course have to be fixed (hatched triangle, Fig. 2b). The triangles of the first column are then fitted with two common vertices between successive triangles (white triangles in Fig. 2b). In the second column, the first and second triangle are fitted with two common vertices also (white triangles in Fig. 2b), but the third (shaded triangle in Fig. 2c) has to be fitted in the triangular hole defined by its neighbours.

Automatic fitting of plane elements has already been discussed by several authors such as Etchecopar (1977) and Cobbold (1979). Each element is fitted in a hole defined by its neighbours in a manner that minimizes voids and overlaps between elements. In the computer program described here a simple algorithm is used to calculate the position of each triangle in order to have a minimum value of the sum of the distances (D) between the vertices of each triangle and those of the triangular hole defined by its neighbours. An example is given in Fig. 2(f), where the triangle ABC is fitted into the triangular hole A′B′C′. Fitting is obtained both by translation and by rigid rotation. The minimum value of D is obtained when its partial derivatives (translation and rotation) are simultaneously equal to zero. To minimize the D value by translation the two centres of gravity of the two triangles must coincide (Etchecopar 1977). To minimize the D value by rotation the following relation must be verified ($\partial D/\partial\alpha = 0$):

$$\begin{aligned} &m\mathrm{A}\cdot m\mathrm{A}'\cdot\sin\alpha + m\mathrm{B}\cdot m\mathrm{B}'\cdot\sin\alpha\cdot\cos\beta \\ &\quad + m\mathrm{B}\cdot m\mathrm{B}'\cdot\sin\beta\cdot\cos\alpha + m\mathrm{C}\cdot m\mathrm{C}'\cdot\cos\gamma\cdot\sin\alpha \\ &\qquad + m\mathrm{C}\cdot m\mathrm{C}'\cdot\sin\gamma\cdot\cos\alpha = 0. \end{aligned}$$

This means that:

$$\tan\alpha = \frac{-(m\mathrm{B}\cdot m\mathrm{B}'\cdot\sin\beta + m\mathrm{C}\cdot m\mathrm{C}'\cdot\sin\gamma)}{(m\mathrm{A}\cdot m\mathrm{A}' + m\mathrm{B}\cdot m\mathrm{B}'\cdot\cos\beta + m\mathrm{C}\cdot m\mathrm{C}'\cdot\cos\gamma)}.$$

The various parameters in the above equations are defined in the caption of Fig. 2(f).

This minimum distance algorithm is used for the fitting of all the triangles that must be fitted to three vertices (shaded triangles, Fig. 2d). In contrast the white triangles (Fig. 2d) are fitted simply by using the two common vertices between these triangles and an average position of their neighbours. The computer program is able to treat successives columns of various length, using only the two vertices and three vertices methods of fitting given above (Fig. 2d). After this initial fitting of all the triangles, an approximate initial state of the surface is obtained. Then additional fitting iterations are done. At this stage almost all the triangles have more than one neighbour and the minimum distance algorithm may thus be used more systematically (Fig. 2e). Each iteration calculates the distance between the vertices of each triangle and those of its hole and sums the distances for all the triangles (ΣD for m vertices). As long as this sum decreases the iterative process continues. As soon as this sum begins increasing (after n iterations), the process is stopped, and the geometry of the layer is considered as the best initial state of the layer (Fig. 2e). The ratio between the ΣD value and the sum of the length of the medians of the triangles (ΣM for m medians) tests the reliability of the unfolding process. A layer folded without change of thickness and well drawn is restored with low $\Sigma D/\Sigma M$ value. In contrast the restoration of a layer badly drawn or the restoration of a layer folded with change of thickness must lead to high $\Sigma D/\Sigma M$ value. The range of values for the fitting indicators $(\Sigma D/\Sigma M) \times 100$ can be discussed by an experimental test of the method.

APPLICATION OF THE PROGRAM TO A PHYSICAL MODEL

As the aim of this application was to test the accuracy of the method, several constraints had to be considered: layer folded without any change of thickness, initial and final geometry of the layer well known, folded surface easy to describe (by successive cross-sections), irregular limits of the surface to model fault boundaries, and simple displacement field to test the results of the program.

To comply with these considerations, the method was tested on a folded sheet of paper. The sheet of paper was first included within 15 sheets of wax (see Odonne & Vialon 1983), then a displacement was imposed along the edge to induce heterogeneous folding (with shortening varying from 11.5 to 17%). The deformed configuration of the paper is given on Fig. 3(a) (arrows indicate the displacement). After deformation the model was cut by five successive cross-sections. These cross-sections were digitized. Then interpolation was carried out between these sections, by using the spline function in the graphics GREG program. Figure 3(c) show that the sections through the interpolated surface match the initial sections fairly well.

The restoration calculated by the program (Figs. 4a & c) is in good agreement with the true initial state (Fig. 3a). The slight variations of width in the restored state (less than 2%) probably results from minor problems of accuracy in the method used to draw the folded surface, and in the fitting process.

Firstly, the cutting of the model may have locally distorted the sheet of paper and the digitization process may have introduced some errors. These two kinds of errors were assumed to be smaller than the size of an element (1%). Secondly, the true initial width of the sheet was considered as regular, since the lateral limits

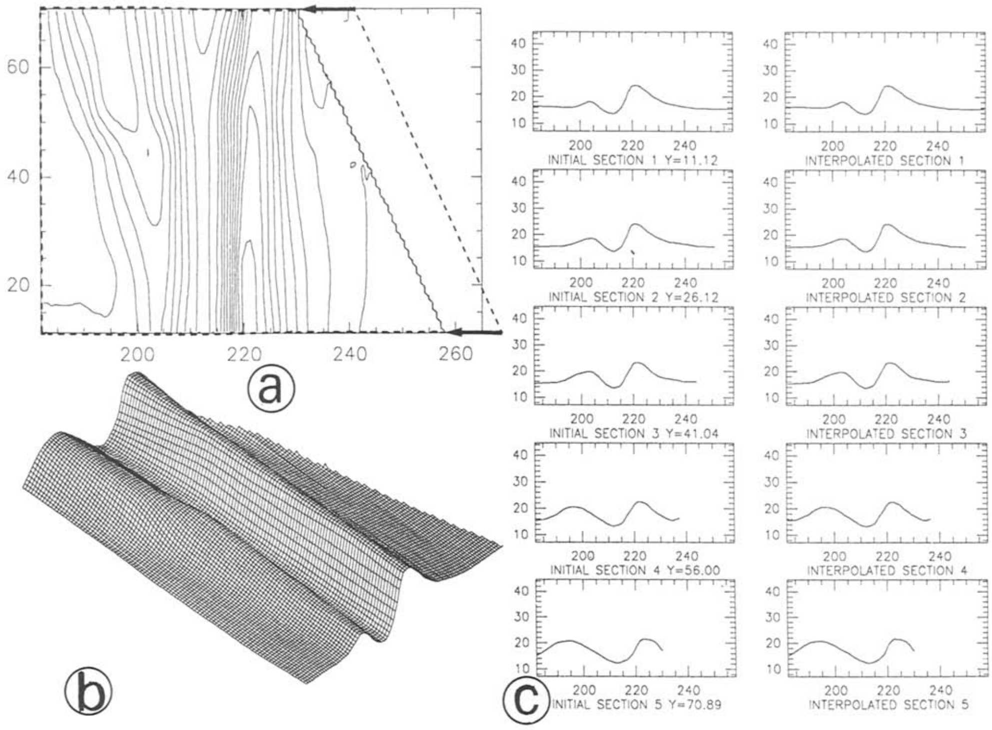

Fig. 3. (a) A representation of the folded sheet of paper embedded in wax, used as a physical model to test the program, showing map view with contours. The initial state is outlined by dashed boundaries, and the displacement from initial to final state by arrows. (b) Perspective view of the deformed sheet. (c) Successive cross-sections through the sheet of paper embedded in wax. The column on the left are the digitized sections and on the right sections are drawn through the interpolated surface.

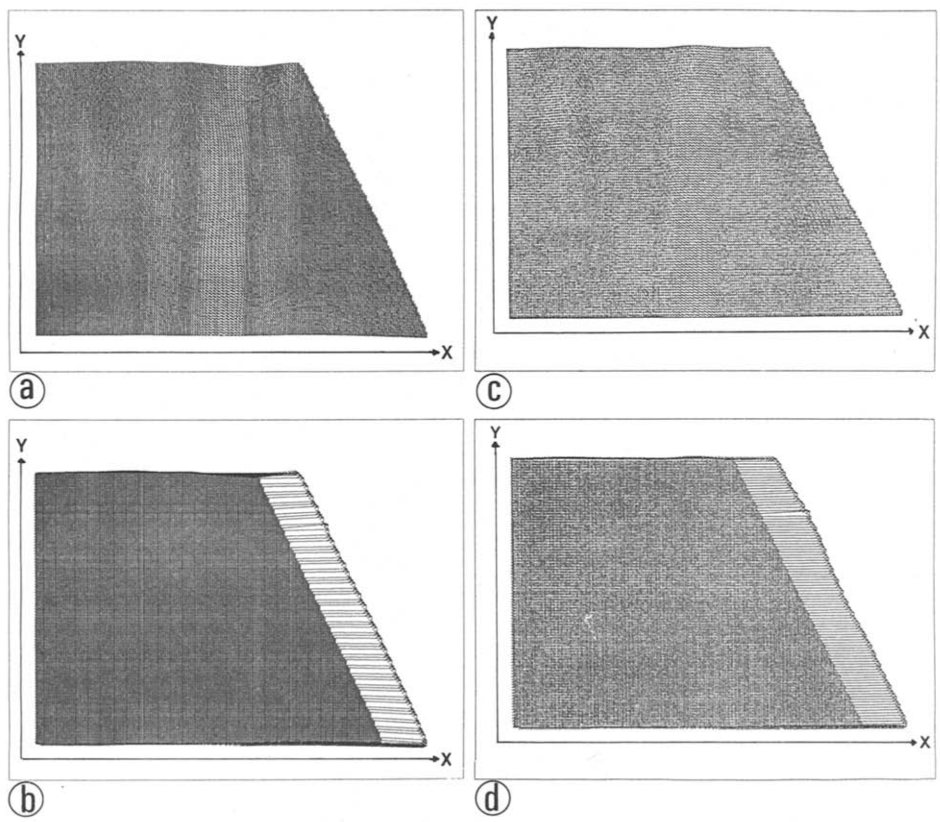

Fig. 4. Map view restoration of the folded sheet of paper using 15,984 triangular elements and with two different sequences of fitting: the first restored column (a & b), or row (c & d), is either parallel to the Y axis (left-hand side, a & b), or parallel to the X axis (bottom side, c & d). The restored state is given in the two cases (a & c) and may be compared with the true initial state (dashed boundaries, Fig. 3a). The total finite displacement of the edge of the sheet, from the initial to the restored state, is given in (b) & (d), and may be compared with the true displacement shown by arrows in Fig. 3(a).

of the sheet were vertical sections. This was not exactly the case in the deformed state since the change of direction and plunge of the folds introduced a difference between the true initial width of the folded sheet and its width on a map view. This difference was not taken into account since it also remains smaller than the size of one element (see initial state Fig. 3a).

Finally two sequences of fitting were tested with restoration beginning either parallel to Y (left first column, Figs. 4a & b) or parallel to X (bottom first row, Figs. 4c & d). A small difference appears between these two sequences of fitting, particularly on the edge farther from the reference axes. The difference of width between the two sequences may very locally reach 1%. The total displacement of the mobile limit is also given correctly both for its direction and for its values (see Figs. 4b & d, and compare with the true displacement in Fig. 3a).

This application on a well drawn surface, folded without change of thickness, also gives experimental values for the fitting indicator $(\Sigma D/\Sigma M) \times 100$. Depending on the sequence of restoration, these values range from 1% (first column perpendicular to the displacement, Figs. 4 a & b) to 1.25% (first row parallel to the displacement, Figs. 4c & d). The first type of sequence is slightly better than the second.

APPLICABILITY TO THE RESTORATION OF NATURAL STRUCTURES

This program may have at least two kinds of application for naturally deformed structures.

(1) The program is useful for testing the restoration of a folded surface limited by irregular boundaries.

In this case the values of the fitting indicator give the reliability of the restoration.

A layer well drawn and folded without change of thickness must be restored with a low fitting indicator value, for example 1% as experimentally tested on the folded sheet of paper. Such a range of values (0.5–1%) has been confirmed by first tests on the restoration of a naturally folded and faulted competent layer, in a region with high density of data (oilfield). A layer folded with change of thickness or a layer folded without change of thickness but badly drawn may also be restored by the computer program, but with high value for the fitting indicator (for example 40% given by the attempt to restore a hemispherical dome). The problem is then the following.

(i) If it is certain (for example by microstructural analysis) that no change occurs in the thickness of the layer during deformation, or that this change is known, the program is useful to help interpretation of a folded surface that is poorly constrained by data. If the surface is not restorable (high fitting indicator value) the shape of the folds must be modified by a trial and error method until a restorable interpretation (with low fitting indicator value) is obtained.

(ii) If the geometry of a layer is perfectly known the program is useful in revealing the zone of change of thickness and in estimating the magnitude of this change.

The fitting indicator can also be estimated for each pair of triangles (triangular element/triangular hole). Distribution values of such indicators are helpful for interpretation by the trial and error method.

(2) The program is also useful for testing the restoration of folded and faulted surfaces.

In this case (Fig. 1) after the restoration and balance of each folded zone (as explained above, see Fig. 1c), all these zones must fit together (Fig. 1d). This fitting is not automatic. Rigid translation and rotation of the restored zones are done with a work station (Bull DPX 1000). Misfitting along the faults implies either local internal deformation or bad drawing of the limits of these faults. As for the preceding application, a trial and error method allows one to estimate either the area change (on a well constrained layer) or to help interpretation of the limits of the faults (on a layer with no change of thickness).

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